OCR
RECOGNISING ACHIEVEMENT

General Certificate of Education
Advanced Supplementary (AS) and Advanced Level

former Oxford and Cambridge modular syllabus

MEI STRUCTURED MATHEMATICS **5508/1**

Mechanics 2

Friday **19 JANUARY 2001** Morning 1 hour 20 minutes

Additional materials:
- Answer paper
- Graph paper
- Students' Handbook

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/ answer booklet.

Answer **all** questions.

Write your answers on the separate answer paper provided.

If you use more than one sheet of paper, fasten the sheets together.

INFORMATION FOR CANDIDATES

The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.

You are advised that an answer may receive no marks unless sufficient detail of the working is shown on the answer paper to indicate that a correct method is being used.

Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise indicated.

This question paper consists of 5 printed pages and 3 blank pages.

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1

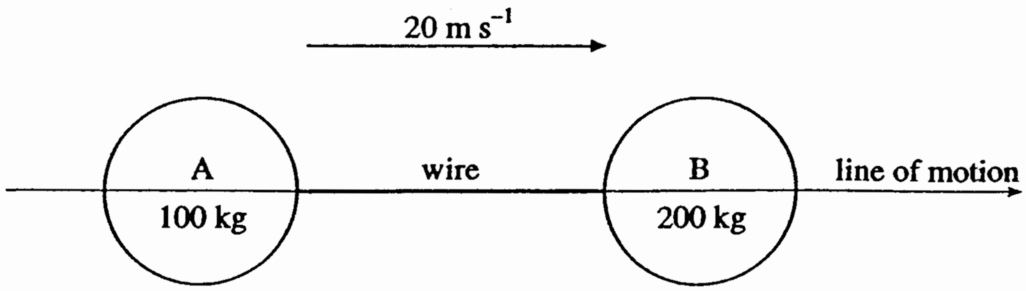


Fig. 1

Two small spacecraft, A with mass 100 kg and B with mass 200 kg, are modelled as moving in the absence of external forces. Both spacecraft are moving at 20 $m\,s^{-1}$ along the same line of motion. They are connected by a light wire in the line of motion, as shown in Fig. 1.

In an attempt to bring the spacecraft together, the tension in the wire is instantaneously increased so that an impulse of magnitude 1100 N s acts on each of the spacecraft.

(i) Show that the new velocity of A is 31 $m\,s^{-1}$ in the original direction of motion and find the new velocity of B. [3]

The spacecraft collide with a direct impact. The coefficient of restitution is $\frac{3}{4}$.

(ii) Show that the velocity of A after the collision is 11.75 $m\,s^{-1}$ in the original direction of motion. [6]

Before the wire becomes taut again, a component of spacecraft A is fired off in the opposite direction to the motion of A. The component has mass 20 kg. The main part of A and the component separate at 70 $m\,s^{-1}$.

(iii) Show that the new velocity of the component is 44.25 $m\,s^{-1}$ in the opposite direction to the motion of A. [4]

The two spacecraft now collide again and coalesce.

(iv) Calculate the final joint velocity of the two spacecraft. [2]

2

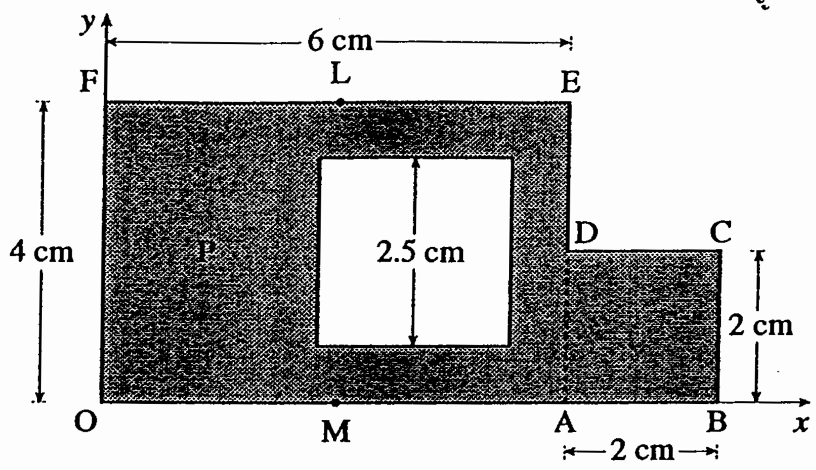


Fig. 2

In this question, coordinates refer to the axes shown in Fig. 2 and the units are centimetres. Answers should be given correct to three significant figures.

A thin, uniform sheet of metal is cut to form the shape shown in Fig. 2. OAEF is a rectangle and ABCD is a square with the dimensions shown. A square of side 2.5 cm and centre (4, 2) has been removed leaving the shaded part P.

(i) Show that the x-coordinate of the centre of mass of P is 3.45, correct to three significant figures, and calculate the y-coordinate. [5]

L and M are the mid-points of EF and OA respectively. The shape P is freely suspended from L.

(ii) Calculate the angle that LM makes with the vertical. [3]

The mass of the square removed is 0.05 kg.

(iii) What vertical force must be applied at O so that when P is freely suspended from L, the line LM is vertical? [4]

The shape is now folded along AD so that the square ABCD is at right angles to OAEF. The x- and y-axes are along OA and OF, as before.

(iv) Calculate the x- and y-coordinates of the centre of mass of P now that it is bent. [3]

3

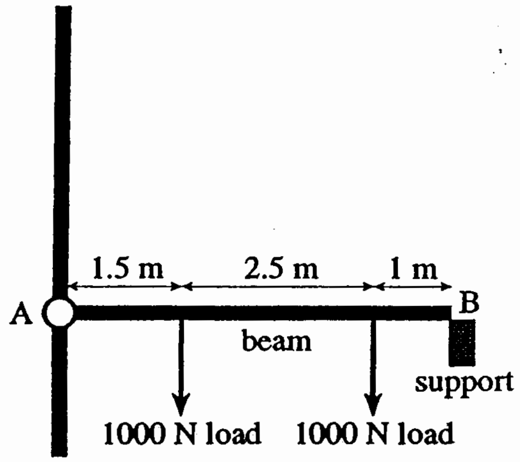


Fig. 3.1

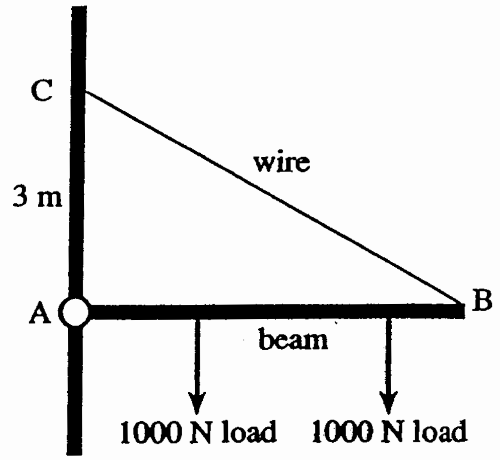


Fig. 3.2

Overhead cables for a tramway are supported by uniform, rigid, horizontal beams of weight 1500 N and length 5 m. Each beam, AB, is freely pivoted at one end A and supports two cables which may be modelled by vertical loads, each of 1000 N, one 1.5 m from A and the other at 1 m from B.

In one situation, the beam is held in equilibrium by resting on a small horizontal support at B, as shown in Fig. 3.1.

(i) Draw a diagram showing all the forces acting on the beam AB. Show that the vertical force acting on the beam at B is 1850 N. [4]

In another situation, the beam is supported by a wire, *instead of the support at* B. The wire is light, attached at one end to the beam at B and at the other to the point C which is 3 m vertically above A, as shown in Fig. 3.2.

(ii) Calculate the tension in the wire. [5]

(iii) Find the magnitude and direction of the force on the beam at A. [6]

4

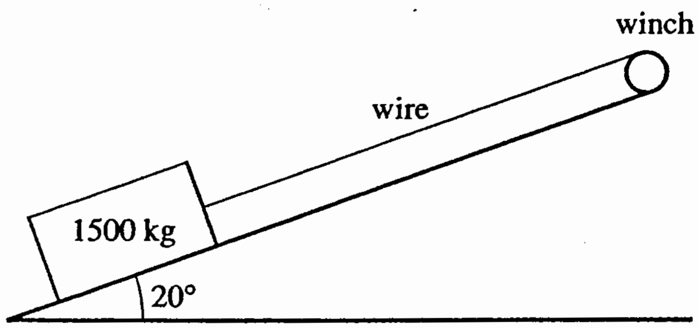


Fig. 4.1

A winch pulls a crate of mass 1500 kg up a slope at 20° to the horizontal. The light wire attached to the winch and the crate is parallel to the slope, as shown in Fig. 4.1.

The crate takes 50 seconds to move 25 m up the slope at a constant speed when the power supplied by the winch is 6 kW.

(i) How much work is done by the tension in the wire in the 50 seconds? [1]

(ii) Calculate the resistance to the motion of the crate up the slope. [4]

(iii) Show that the coefficient of friction between the crate and the slope is 0.50 (correct to two significant figures). [3]

The winch breaks down and the crate is then *pushed* up the slope by a mechanical shovel by means of a constant force of 16 000 N inclined at 15° to the slope, as shown in Fig. 4.2. You may assume that the crate does not tip up.

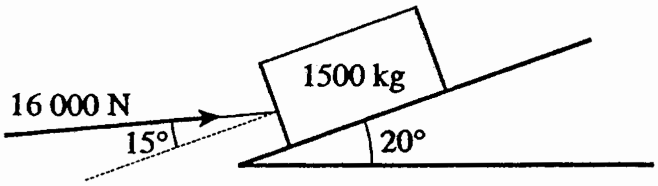
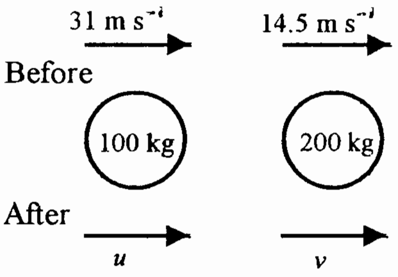
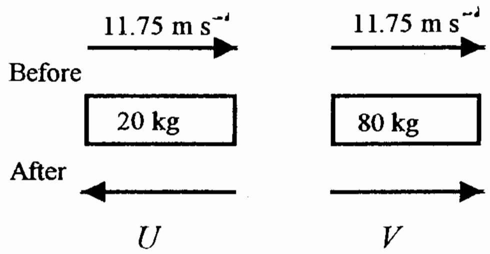


Fig. 4.2

(iv) Calculate the distance travelled by the crate up the slope as it speeds up from rest to 2.5 m s^{-1}. [You may assume the coefficient of friction between the crate and the slope is exactly 0.5.] [7]

Mark Scheme

1. (i)	For A		
	$\rightarrow 100 \times 20 + 1100 = 100v$	M1 PCLM applied to A	
	so 31 m s^{-1} in original direction	E1 (Accept no direction given)	
	For B		
Either	$300 \times 20 = 31 \times 100 + 200v'$		
Or	$200 \times 20 - 1100 = 200v'$		
	$v' = 14.5$ so 14.5 m s^{-1} in original direction	F1 (Direction must be established) [Allow without M1]	[3]
(ii)		B1 (Award if solution correct)	
PCLM			
	$300 \times 20 = 100u + 200v$	M1 PCLM	
	$u + 2v = 60$	A1 Any form	
	NEL		
	$\frac{v-u}{14.5-31} = -\frac{3}{4}$	M1 NEL	
	$4v - 4u = 49.5$	A1 Any form	
	Solving gives $u = 11.75$		
	so 11.75 m s^{-1} in original direction	E1 (Accept no direction specified)	[6]
(iii)			
PCLM	$100 \times 11.75 = -20U + 80V$	M1 Use of PCLM on A. Accept wrong sign A1	
	Also $U + V = 70$	B1 Signs must be consistent with PCLM	
	Solving gives $U = 44.25$ so 44.25 m s^{-1} in opposite to orig direction	E1 Direction must be explicit and justified	[4]
(iv)	PCLM overall		
	$\rightarrow 300 \times 20 = -20 \times 44.25 + 280W$	M1 Allow wrong masses	
	$W = 24.589$.. so about 24.6 m s^{-1} in original direction	A1 Accept no direction specified	[2]
		[Total 15]	

2. (i)

$$24\begin{pmatrix}3\\2\end{pmatrix}+4\begin{pmatrix}7\\1\end{pmatrix}-6.25\begin{pmatrix}4\\2\end{pmatrix}=21.75\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

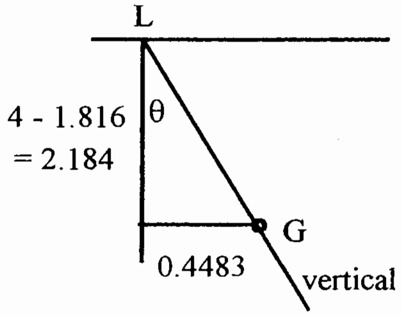
M1 Appropriate method

B1 All masses correct
B1 At least two sets of coordinates correct on LHS

so (3.44827..., 1.81609..)
giving (3.45, 1.82), (3 s.f.)

E1 A1 [5]
[If separate components used, award the 2nd B1 if all the terms correct for 1 cpt]

(ii)



$\tan\theta = \frac{0.4483}{2.1839}$ M1 Correct angle identified

$\theta = 11.600...°$ B1 Use of **their** $\bar{y}$ and given $\bar{x}$, including 4 – **their** 1.816.

so 11.6°, (3 s.f.) A1 (Accept 11.7° found if rounded values used) [3]

(iii) 6.25 cm^2 has mass 0.05 kg
so density is 0.008 kg cm^{-2} B1 May be implied

So $21.75 \times 0.008g \times 0.4483 = 3F$ M1 Moments used (allow mass instead of weight)
B1 Use of weight

$F = 0.2547...$ so about 0.255 N A1 [4]

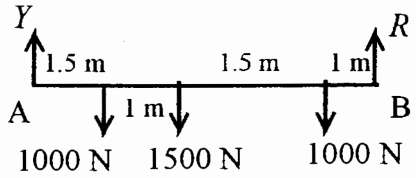
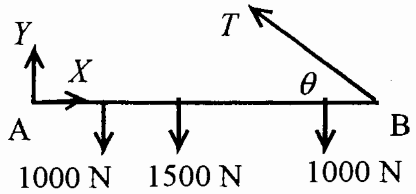
(iv) y coordinate unchanged so 1.82, (3 s.f.) B1 Accept without comment

$21.75\bar{x} = 75 - 4 \times 7 + 4 \times 6$ M1 Attempt to deal with the fold. Masses must be consistent. Total mass is **their** 21.75 from (iii).

$\bar{x} = 3.2643...$ so 3.26, (3 s.f.) A1 cao [3]

[Total 15]

3. (i)

	B1 Diagram. Ignore horizontal forces. Lengths not required.
A (moments) $1.5\times1000+2.5\times1500+4\times1000=5R$	M1 Moments about A. Must show terms. B1 Moment of weight of beam
$9250=5R$ $R=1850$ so 1850 N vert upwards	E1 Direction not required [4]
(ii) 	B1 Diagram. Must have X and Y or resultant. Award if method correct with correct signs and angles.
$\tan\theta=\frac{3}{5}$, $\sin\theta=\frac{3}{\sqrt{34}}$	B1 Either seen or implied
Either A (moments) $9250=5T\sin\theta$	M1 Moments about A B1 Dealing with moment of T ($\sin\theta$ need not be evaluated)
$T=3595.75..$ so 3596 N (4 s.f.)	A1 Accept 2 s.f. or better. [5]
Or Using (i) $T\sin\theta=1850$	M1 B1 for $T\sin\theta$.
$T=\frac{1850\sqrt{34}}{3}=3595.75...$ so 3596 N (4 s.f.)	A1 Accept 2 s.f. or better.
(iii) Resolve horizontally and vertically	M1 Both attempted or resolve + moments
$\uparrow$ $Y+1850=3500$ so $Y=1650$	B1
$\rightarrow$ $X-T\cos\theta=0\Rightarrow X=1850\cot\theta=\frac{9250}{3}$	B1 Accept any form
	M1 correct method for magnitude **or** direction
$\sqrt{1650^2+(\frac{9250}{3})^2}=3497.06..$ so 3497 N (4 s.f.)	A1 cao. Accept 2.s.f. or better
angle is $\arctan(\frac{3\times1650}{9250})=28.152...$ so about 28.2° above horizontal	A1 cao Accept 2 s.f. or better. Must specify or imply direction with horiz or vertical; accept a clear diagram. [6]
	[Total 15]

	Solution	Marks	
4. (i)	$50 \times 6000 = 300\,000$ J	B1	[1]
(ii)			
Either	Considering the total work done	M1 Must have weight term	
	$25F + 1500g \sin 20 \times 25 = 300000$	B1 B1 Each term on LHS	
	$F = 6972.30..$ so 6972 N (4 s.f.)	A1 Accept 2 or 3 s.f.	
Or	Tension in wire is $\frac{6000}{0.5} = 12000$	M1 Use of $T = \frac{P}{v}$, explicit or implied. A1	
	so $12000 = F + 1500g \sin 20$	M1 Must have weight term	
	$F = 6972.30..$ so 6972 N (4 s.f.)	A1 Accept 2 or 3 s.f.	[4]
(iii)	$\mu = \frac{6972.30...}{1500 \times 9.8 \times \cos 20}$	M1 Use of $F = \mu R$ B1 Correct R	
	$= 0.50474...$ so 0.50 (2 s.f.)	E1	[3]
(iv)	New $F_{max} = 0.5(1500g \cos 20 + 16000 \sin 15)$	M1 Attempt to find new F	
	$= 8977.293....$	A1 Need not be evaluated	
Either	Using work-energy	M1 Equating WD by shovel to WD against friction + GPE + KE	
	$16000 \cos 15x$ $= 8977.293...x + 1500g \sin 20x + \frac{1}{2} \times 1500 \times 2.5^2$	B1 B1 for KE term B1 Other terms (FT for friction)	
	$x = 3.233...$ so 3.23 m (3 s.f.)	A1 cao	
Or	Using N2L	M1 Use of N2L **and** 'uvast'	
	$16000 \cos 15 - 8977.293.. - 1500g \sin 20 = 1500a$	M1 Accept no weight term. All other forces present.	
	$a = 0.966549...$	A1 Need not be evaluated. FT for friction.	
	$2.5^2 = 0 + 2as$	B1 (or other valid sequence of '*uvast*''. FT wrong a.	
	$x = 3.233...$ so 3.23 m (3 s.f.)	A1 cao	[7]

[Total 15]

Examiner’s Report

Mechanics 2 (5508)

General Comments

There was a wide variation in the standard of the answers to each of the questions but many complete answers were seen to each question and most candidates were able to make some progress with every question.

Many candidates created problems for themselves by their failure to draw adequate diagrams; this was a particular handicap in Q.1 and Q.3. Also, there were many examples of candidates using, say, F for every unknown force in a question and then either selecting the wrong value later or even eliminating this 'common' variable.

Many candidates failed to give due consideration to the accuracy of their final answer. Unless otherwise specified, two or three significant figure accuracy for the answer will be accepted provided that the accuracy of the working supports this. Candidates often work to, say, two figures and then quote their answers to two or even more figures; this type of error may be penalised.

Comments on Individual Questions

Question 1 (Impulse, momentum and the direct impact of bodies)

Although most candidates could establish the given result in part (i), many failed to establish the velocity of B. In some cases this was simply the omission of the final direction of B; a clear statement about which direction was positive anywhere in the part was accepted. In other cases a method was stated or implied where the internal nature of the impulse was not appreciated and the linear momentum of the whole system was increased by 1100 N s. For some candidates this was a simple sign error, for others it was clearly a misunderstanding of the situation.

The techniques required in part (ii) were usually well understood but many were unable to establish the given result because of their error in part (i); few candidates seemed able to go back and correct their mistake. A number of candidates were unable to use Newton's experimental law correctly, usually because of sign errors in the absence of a diagram.

Part (iii) seemed to be well understood but there were many sign errors caused by the lack of a clear diagram. A number of candidates were unable to use correctly the information about the velocity of separation, often because they tried to incorporate this information into their equation for the conservation of linear momentum assuming that one of the bodies had a speed of 70 $m\,s^{-1}$.

Few candidates were successful with part (iv). The most common mistakes were to use the wrong masses or the value of a velocity for A or B that had subsequently changed. Very few candidates solved the problem neatly by arguing that the final linear momentum could be found immediately by considering the initial value and the momentum of the part lost to the system.

(i) 14.5 $m\,s^{-1}$ in the original direction of motion; (iv) 24.6 $m\,s^{-1}$ (3 s. f.).

Question 2 (Centre of mass and the use of moments)

Parts (i) and (iv) were generally done very well with most candidates understanding what was required. A number of candidates divided the lamina into many parts, often unlabelled, and so made their analysis unnecessarily complicated.

Part (ii) was generally done quite well but a large minority of candidates did not draw a suitable diagram to locate the angle required and subsequently either found the wrong angle or made mistakes when calculating the dimensions of their triangle.

Part (iii) was not done as well as the rest of the question. Many candidates failed to use the information about the density to find the weight, many omitted forces from their diagrams (if present) and so did not realise that they must take moments. Others wrote down incorrect equations because they took moments about, say, A or O and neglected the vertical force at L.

(i) (3.45, 1.82) (3 s.f.), (ii) 11.6° (3 s.f.), (iii) 0.255 N (3 s.f.), (iv) (3.26, 1.82) (3 s.f.).

Question 3 (Moments applied to a beam)

The solutions to this question were not as good as those to Q.1 and Q.2, usually because of the lack of correct diagrams.

In part (i), many candidates did not draw a diagram. Some omitted the reaction at A and others the weight of the beam. Most understood the need to take moments.

In part (ii), many of the candidates clearly understood that all they had to do was to replace the vertical force at B found in part (i) with the vertical component of the tension in the wire and readily obtained the correct solution. Others with less clear understanding wrote down equations that mixed forces and moments of forces. There were many examples of rounding errors with many candidates finding the angle correct to two significant figures and then giving their final answer to three, four or even five significant figures.

Part (iii) was frequently not done well. The most common mistakes were due to lack of proper organisation. Many candidates attempted the solution without a diagram so that the sense of the force components was unclear and confusion invariably followed. For instance, candidates often failed to distinguish between the horizontal component of the tension acting at B and the horizontal component of the force acting at A.

(ii) 3600 N (3 s.f.), (iii) 3500 N at 28.2° upwards to AB (3 s.f.).

Question 4 (Power, work and energy applied to a crate on a slope)

This question was not done well by many candidates.

Many candidates wrote down numerical expressions in parts (i) and (ii) without any attempt to say what they were supposed to be; in some cases there was an apparent failure to recognise or understand what these quantities were and to know when the question had been answered. This confusion seemed to be caused by a lack of appreciation of the technical meanings of *power*, *work*, *energy* and *force*; these quantities were frequently combined in inappropriate ways.

Most candidates recognised what had to be done in part (iii).

Few candidates completed part (iv) successfully although many understood the principles involved. The most common error was to fail to realise that the normal reaction and hence limiting friction had changed in this new situation. Omission of one or more of the forces was also quite common. Most candidates who attempted this part used Newton's second law but, as always, those who used a work-energy method were more often successful.

(i) 300 000 J, (ii) 6970 N (3 s.f.), (iv) 3.23 m (3 s.f.).